Density Determinations and Various Methods to Measure Volume

GOAL AND OVERVIEW

This lab provides an introduction to the concept and applications of density measurements. The densities of brass and aluminum will be calculated from mass and volume measurements. To illustrate the effects of precision on data, volumes will be determined by three different methods: geometrically (measuring lengths); water displacement; and pycnometry. The composition of a mixed brass-aluminum cylinder and the volume of empty space within a hollow cylinder will also be found.

Objectives of the Data Analysis

- Determine volume by three different methods
- Use measured volumes and masses to calculate densities
- Use the relationship between mass, volume, and density to find desired unknown quantities
- Evaluate results using error analysis

SUGGESTED REVIEW AND EXTERNAL READING

- Data analysis introduction, textbook information on density

INTRODUCTION

The density, ρ, of an object is defined as the ratio of its mass to its volume. Density can be useful in identifying substances. It is also a convenient property because it provides a link (or conversion factor) between the mass and the volume of a substance.

$$\rho = {}^{m}/_{V} \tag{1}$$

Mass and volume are extensive (or extrinsic) properties of matter - they depend on amount. Density, an intensive (or intrinsic) property, is a kind of "heaviness" factor. In macroscopic terms, density reflects how much mass is packed into a given three-dimensional space. Typically, densities are reported g/ml or g/cm^3 (which are equivalent because 1ml $\equiv$ 1cm^3). Experimentally, mass and volume measurements are required to calculate density.

Masses are measured on electronic balances. Pan balances, which are accurate to ± 0.01 g, are used for quick measurements where greater precision is not required. Analytical balances (accurate to ± 0.0001 g) are used for more precise measurements.

Volume is an amount of space, in three dimensions, that a sample of matter occupies. The number and the phase of the molecules in the sample primarily determine the volume of a substance. Volume will be measured in many ways in this course, but the units are usually milliliters (mL) or cubic centimeters (cm^3). Methods for determining or delivering precise volumes include volumetric pipets and pycnometers; less precise methods include burets, graduated cylinders, and graduated pipets.

In this experiment, you will measure masses and volumes to determine density. Four different metal cylinders are investigated.

In parts 1-3, three different methods are used to find volume of two solid metal cylinders (Al and brass). Each method has its own degree of precision.

(i) volume by geometry

(ii) volume by water displacement

(iii) volume by pycnometry (mass-based)

In parts 4-5, one method for volume determination is used to find:

(i) the volume of a void inside a hollow cylinder; and,

(ii) the percent composition of a mixed-metal cylinder.

Volume by geometry

A cylinder is a standard geometric form. In this case, you can measure the dimensions of the cylinder and apply the formula to calculate its volume.

$$V = \pi \left(\frac{d}{2}\right)^2 l = \frac{\pi}{4} d^2 l \quad \text{where } d = \text{diameter and } l = \text{length.} \tag{2a}$$

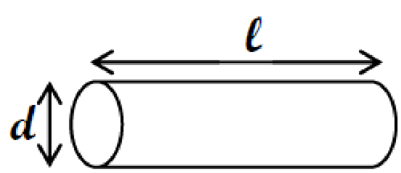

Figure 1

Density would be calculated in one step to minimize rounding errors:

$$\rho = \frac{m}{V} = \frac{4m}{\pi d^2 l} \tag{2b}$$

The uncertainty in the volume must be determined by error propagation. Mass, length, and diameter measurements contribute to the overall uncertainty.

$$\sigma_\rho = \pm \rho \left[\left(\frac{\sigma_d}{d}\right) + \left(\frac{\sigma_d}{d}\right) + \left(\frac{\sigma_l}{l}\right) + \left(\frac{\sigma_m}{m}\right) \right] \tag{2c}$$

Volume by water displacement

For less defined shapes, volume can be determined by water displacement. Volumes of liquids such as water can be readily measured in a graduated cylinder.

To use the water displacement method, an object (in this case, a small metal cylinder) is inserted into a graduated cylinder partially filled with water. The object's volume occupies space, displacing liquid and raising the water level. The difference between the two volumes, before and after the object was inserted, is the object's volume.

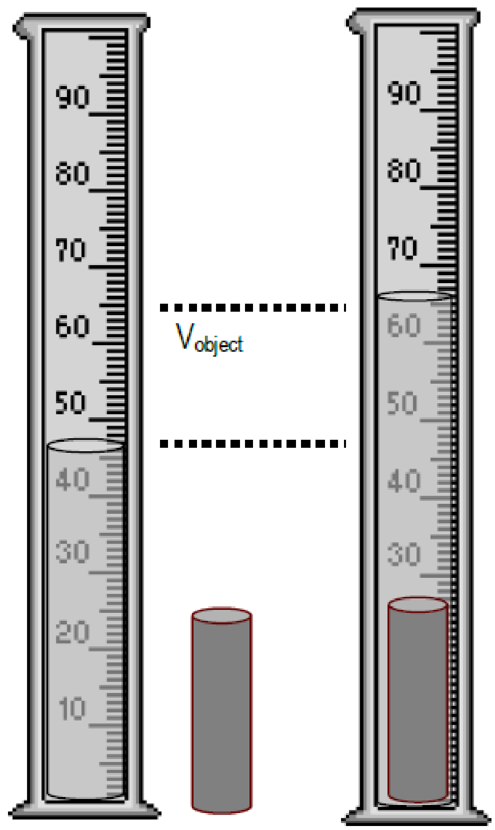

Figure 2

$$V_{cyl} = V_{final} - V_{initial} = V_{water+cyl} - V_{water} \tag{3a}$$

The uncertainty of the volume is based on the two volume readings.

$$\sigma_{V_{cyl}} = \sigma_{V_{final}} + \sigma_{V_{initial}} \tag{3b}$$

The density is calculated using m/V. There is not a straightforward way to find density in one step (as with geometry).

The uncertainty in the density would be given by:

$$\sigma_\rho = \pm\rho\left[\left(\frac{\sigma_V}{V}\right) + \left(\frac{\sigma_m}{m}\right)\right] \tag{3c}$$

Volume by pycnometry

Pycnometry is a technique that uses the density relationship between volume and mass, and the vessel used is called a *pycnometer* .

To perform pycnometry measurements, the mass of the cylinder and the mass of a flask filled with water to a mark (A, Fig. 3) are recorded. The cylinder is then inserted into the flask. Water is displaced when the cylinder is inserted. The volume of water displaced is removed by pipet, thereby restoring the water level to the mark (B). The combined mass of the flask, remaining water, and cylinder is then measured.

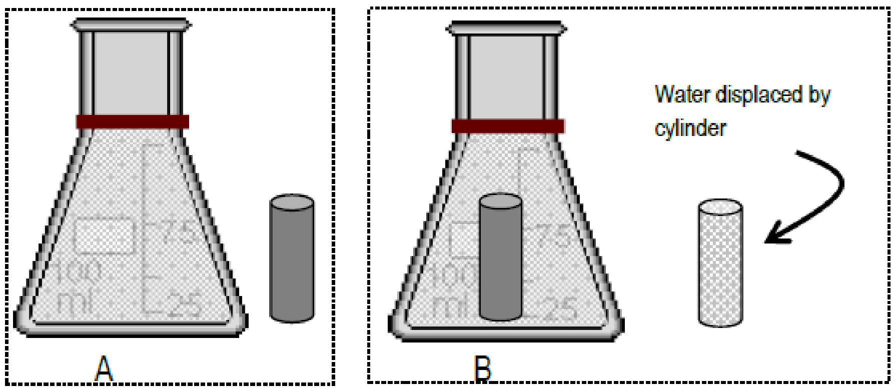

Figure 3

The sums of the masses before and after are equal. The $mass_A$, the $mass_B$, and the $mass_{cylinder}$ were all measured on the balance. There is only one unknown in the equation - the mass of the displaced water.

$$\begin{aligned} mass_A + mass_{cylinder} &= mass_B + mass_{displacedwater} \\ mass_{displacedwater} &= mass_A + mass_{cylinder} - mass_B \end{aligned} \tag{4a}$$

The volume of water removed is equal to the volume of the cylinder. $Mass_{water}$ can be converted to volume using the density of water.

$$V_{displacedwater} = V_{cylinder} = {}^{mass_{displacedwater}} /_{density_{water}} \tag{4b}$$

The density of the cylinder is calculated using m_{cyl}/V_{cyl}.

The uncertainty calculation requires a few steps and assumptions. The volume of the cylinder was equal to the volume of the water. V_{water} was based on the three mass measurements - the mass of the cylinder, of A, and of B.

The uncertainty in mass$_{cylinder}$ comes from the balance reading.

The uncertainty associated with mass$_A$ and mass$_B$ depends on your ability to precisely adjust the level of the water to the mark at the exactly same place every time (calibration). By repeatedly filling the flask to the mark and taking the mass readings, the average mass of A and the standard deviation (the fluctuation in the mass due to variations in the exact liquid level) can be found.

$$\langle m_A \rangle = \frac{m_{A,trial1} + m_{A,trial2} + m_{A,trial3} + \ldots}{\#trials} \tag{4c}$$

$$\sigma_{m_A} = \pm\sqrt{\frac{(\langle m_A \rangle - m_{A,trial1})^2 + (\langle m_A \rangle - m_{A,trial2})^2 + \ldots}{\#trials - 1}} \tag{4d}$$

Assume the uncertainty in the mass of both A and B is the same: m$_A$ $\pm$ σ_{mA}; m$_B$ $\pm$ σ_{mA}.

The uncertainty in the mass of water displaced is determined by error propagation:

$$\sigma_{m_{water}} = \sigma_{m_A} + \sigma_{m_B} + \sigma_{m_{cyl}} = \sigma_{m_A} + \sigma_{m_A} + \sigma_{m_{cyl}} \tag{4e}$$

The density of water at room temperature is known quite precisely and is assumed to contribute negligible error (see table at the end of the lab), so dividing $\sigma_{m,water}$ by the density of water to give $\sigma_{V,water}$ is adequate. Since $\sigma_{V,water} = \sigma_{V,cyl}$, the uncertainty in the density can be determined.

$$\sigma_\rho = \pm\rho\left[\left(\frac{\sigma_V}{V}\right) + \left(\frac{\sigma_m}{m}\right)\right] \tag{4f}$$

You will use pycnometry in parts 4 and 5 to determine the volume and/or density of a hollow cylinder and of a mixed cylinder.

Volume of a void inside a hollow cylinder

A hollow cylinder has an empty space inside.

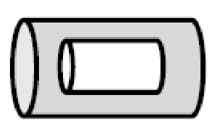

Figure 4

The volume of the cylinder is comprised of the volume of metal and the volume of the void inside.

$$V_{cyl} = V_{metal} + V_{void} \Rightarrow V_{void} = V_{cyl} - V_{metal} \tag{5a}$$

V_{cyl} is determined by pyncometry. The volume occupied by the metal can be determined using the mass of the cylinder (which is due to only the metal, not the void) and the density of the metal, which was determined previously in the lab (either Al or brass, depending on the cylinder). Use the value for density that is closest to the literature values - 2.70 g/cm^3 for Al; between 8 and 9 g/cm3 for brass.

$$V_{metal} = \frac{m_{cyl}}{\rho_{metal}} \text{ No error propagation is required} \tag{5b}$$

Percent composition of a mixed cylinder

The total mass of the cylinder, m_{cyl}, is the sum of the mass of Al and brass (m_{Al} + m_{brass}). In terms of fractional composition, this would be $X m_{cyl}$ and (1 - X)m_{cyl}, respectively, where X is the Al fraction and (1-X) is the brass fraction (the remainder).

The cylinder volume is determined by pycnometry and is the sum of the volumes of the two metals:

$$V_{cyl} = V_{Al} + V_{brass} \tag{6a}$$

Replace each volume by its mass divided by its density using V=m/ρ:

$$V_{cyl} = \frac{m_{Al}}{\rho_{Al}} + \frac{m_{brass}}{\rho_{mass}} \tag{6b}$$

Replace the masses by the equivalent expressions in terms of X and m_{cyl}:

$$V_{cyl} = \frac{X m_{cyl}}{\rho_{Al}} + \frac{(1-X) m_{cyl}}{\rho_{brass}} \tag{6c}$$

Divide through by m_{cyl} and replace V_{cyl}/m_{cyl} with $1/\rho_{cyl}$:

$$\frac{1}{\rho_{cyl}} = \frac{X}{\rho_{Al}} + \frac{(1-X)}{\rho_{brass}} \tag{6d}$$

Collect terms on the right-hand side that contain X:

$$\frac{1}{\rho_{cyl}} = X\left(\frac{1}{\rho_{Al}} - \frac{1}{\rho_{brass}}\right) + \frac{1}{\rho_{brass}} \tag{6e}$$

Solve for ***X*, the mass fraction of aluminum** in the mixed cylinder.

$$X = \frac{\left(\frac{1}{\rho_{cyl}} - \frac{1}{\rho_{brass}}\right)}{\left(\frac{1}{\rho_{Al}} - \frac{1}{\rho_{brass}}\right)} \tag{6f}$$

This is the equation to use. Density of the cylinder is found by pycnometry. The densities of Al and brass have already been determined.

When finding X:

a Calculate each fraction in the equation, then the differences, and then the final ratio.

b Use the densities of brass and aluminum determined experimentally.

c Find X, and use X to determine the mass fraction of brass in the mixed cylinder, 1 - X. X has a range of possible values from zero to one (0 - 100%). If your mixed cylinder's density is between that of aluminum and of brass, you should calculate a percent of aluminum that makes sense. For example, if the mixed cylinder has a density near that of Al, X should be near one.

EQUIPMENT LIST

cylinders: brass, aluminum, mixed brass/aluminum, and hollow

Vernier caliper

50 mL Erlenmeyer flask, 100 mL graduated cylinder, 400 mL beaker

lab marker

Pasteur pipet

thermometer

PROCEDURE

NOTE: IF YOU WORK WITH ANOTHER SET OF PARTNERS, MAKE SURE YOU RECORD ALL DATA. YOU WILL NOT BE ABLE TO COMPLETE THE DATA ANALYSIS IF YOUR DATA TABLES ARE INCOMPLETE. ALSO CHECK THAT DATA MAKES SENSE.

Parts 1-3. Density of aluminum and brass cylinders using three different methods of volume measurement

Part 0: Measure metal cylinder masses.

1 Obtain four cylinders - brass, aluminum (solid cylinders marked S), hollow (marked H), mixed brass/aluminum (marked P for "plugged"). Return cylinders to the stockroom at the end of lab.

2 Record the cylinders' numbers.

3 Record the masses of the cylinders on the analytical balance to the 0.0001 g (the uncertainities in your cylinders' masses are ±0.0001 g). You will use these masses throughout the experiment.

Part 1: Volume by Geometry

1 Measure the diameter and length of each cylinder using the Vernier calipers. Your TA will help you if you need it. Record the values to the 0.01 cm (each measurement is ± 0.01 cm).

2 Determine the density of the cylinder. Find the uncertainity using error propagation.

Part 2: Volume by Displacement

1 Put enough water to cover the metal cylinder into a 100-mL graduated cylinder and record the volume. The graduated cylinder is not very precise; readings will be ± 0.5 mL (the digit in the tenths place will either be a 5 or a 0).

2 **Carefully** slide the metal cylinder down the side of the graduated cylinder into the water. Tossing it in can break the bottom of the graduated cylinder.

3 With the metal cylinder completely submerged, record the new volume reading (to ± 0.5 mL).

4 Determine the volume of the metal cylinder. Calculate the uncertainty in your volume using error propagation.

5 Determine the density of the cylinder. Calculate the uncertainty using error propagation.

Part 3: Volume by Pycnometry

1 Fill a 400 mL beaker with water and *measure its temperature.* Use this water throughout the experiment. Assume that the density of water makes a negligible contribution to the overall uncertainty in the values calculated.

2 Make your pycnometer.

- **a** Draw a ring midway up the neck of a 50 mL Erlenmeyer flask with a waterproof marker or wax crayon, as shown below.
- **b** Invert the flask on the table; hold marker on top of something solid; and, rotate the flask while marking the neck at a constant height.

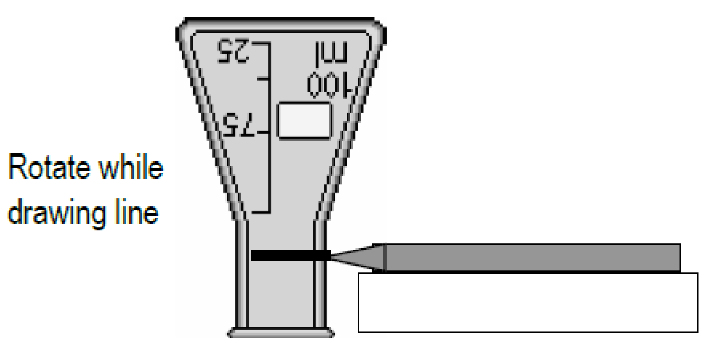

Figure 5

3 Calibrate your pycnometer. How well can you adjust the water's meniscus to the *top* of the line drawn? Precise filling to that mark increases reproducibility (and data quality). Practice with the pycnometer before making measurements. Your TA will demonstrate.

The pycnometer filled with water to the mark is called 'A'.

a Use a disposable pipet to add and remove drops of water to adjust the meniscus to the top of the line.

b Record the mass of the flask and water. No drops should appear on the neck of the flask above the water line.

c Pour out the water into your 400-mL beaker; refill to the mark; reweigh.

d Repeat step c until you have three similar values for the mass of 'A'.

e Determine the average m_A and its standard deviation (σ_{mA}). The standard deviation, σ_{mA}, reflects your ability to reproducibly fill the pycnometer to the same place every time you use it; σ_{mA} is the uncertainity of the pycnometer and should be read with the average mass of 'A' as well as the mass of 'B' (parts 3, 4, and 5).

f *If you share data with another set of students, make sure to record their calibration data as well.* You must use the correct calibration result with the appropriate data. In your lab notebook, label which cylinders go with which pycnometer calibration.

4 Indirectly measure the mass of water displaced by your solid cylinders. The pycnometer containing the metal cylinder with water filled to the mark is called 'B'.

a Carefully insert a metal cylinder, fill with water to the mark, and record the mass (the flask with water and cylinder).

b Repeat filling and weighing several times until the data appears reproducible.

c Calculate the mass of the water removed. Convert this mass to volume by dividing by the density of water (use a precise value, specific to the water's temperature). This volume equals the volume of the metal cylinder.

Calculate the uncertainty in the mass of water removed using error propagation. Convert this mass to volume units by dividing by the density of water (use a precise value, specific to the water's temperature). This value equals the uncertainity in the volume of the metal cylinder.

e Repeat with the other cylinders as instructed.

5 Determine the density of each cylinder. Include the uncertainties.

Please do not throw the metal cylinders away. Please return them to the reagent bench. Please put disposable pipets (and any broken glassware) in broken glass containers, not the trash can. You will lose points for inappropriate disposal.

Part 4: Determine Void Volume in a Hollow Cylinder by Pycnometry

1 For the hollow cylinder, record identity of metal of the hollow cylinder (either aluminum or brass). You recorded its mass at the beginning of the experiment.

2 Insert the cylinder into the pycnometer; remove the water above the line, and record the new mass.

3 Determine the volume of the cylinder and calculate the volume of the void. *No error propagation.*

Part 5: Mass Fraction of Al and Brass Determination for Mixed-metal Cylinder by Pycnometry

1 You recorded its mass of the mixed metal cylinder at the beginning of the experiment.

2 Insert the mixed Al/brass cylinder into the pycnometer, remove the water above the line, and record the mass.

3 Determine the density of the cylinder and the mass fractions of Al and of brass (X and 1-X, respectively). *No error propagation.*

REPORTING RESULTS

Complete your lab summary or write a report (as instructed).

Results / Sample Calculations

Masses and volumes for solid cylinders by each method

%error relative to literature values (ρ_{Al} = 2.70 g/cm^3; ρ_{Al} = 8.44 g/cm^3, depending on the alloy's composition)

Void volume

Mass fraction

Error analysis for parts 1-3

Discussion

What you found out (refer to results tables) and how for all 5 parts

What were the major experimental sources of error?

Compare the three methods used to determine volume - which method was more accurate and why? Which was most precise?

What could be done to improve the precision in any or all of the methods?

How does the instrument error compare to standard deviation error?

Review Questions

Whole degrees are listed down the left hand side of the table[1], while tenths of a degree are listed across the top. So to find the density of water at 5.4 °C, find the whole degree by searching down the left hand column to '5'. Then slide across that row to '0.4'. The density of water at 5.4 °C is 0.999957 g/mL.

[1] images/figure6.png

